A SPICE MODEL FOR MULTIPLE COUPLED MICROSTRIPS AND OTHER TRANSMISSION LINES

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ABSTRACT

A general multiple coupled line model compatible with standard CAD programs, such as SPICE, is presented. It is shown that the model can be used to help analyze and design coupled line (e.g., microstrip) circuits with linear, as well as nonlinear/time varying terminations, and to help study the pulse propagation characteristics in high speed digital and optical circuits.

INTRODUCTION

A considerable amount of work has been done on the properties and applications of multiple coupled distributed parameter systems, including coupled transmission lines. The general solutions for the normal mode propagation constants, eigenvectors, impedances, and the network functions characterizing the 2n port for a coupled n line structure are available in a matrix form [1-8]. In addition, for a smaller number of lines (e.g., two, three, or four lines) these properties can be expressed in an explicit closed form [e.g., 8-10]. Circuits can be analyzed and the design procedures can be formulated by utilizing the 2n port impedance, admittance, or general circuit parameters with the given terminations at the various ports of the structure. Dedicated computer programs for computing the scattering parameters and other properties of a terminated multiport are available for the analysis of such structures. Based on the solution of the coupled line equations, accurate models have been derived primarily for the case of coupled pairs of symmetric lines for microwave circuits and for multiple coupled lines for the analysis of the pulse propagation characteristics of the interconnections in digital circuits.

The model for the coupled pair of symmetrical lines consists of two uncoupled lines and coupling transformers at the input and the output ports and is now available in many microwave circuit design programs. For the case of general multiple coupled line structures, the general coupled line analysis has led to the model consisting of uncoupled lines coupled at the input and the output ends by a congruent transformer bank [1]. This model has been modified by using the method of characteristics, and has been applied to the case of the interconnections in high speed digital circuits that are best represented as coupled transmission lines [11,12]. All these models, together with the ones presented here, are mathematically identical in that they all represent the exact solution of the coupled transmission line equations.

In this paper, a multiple coupled line model consisting of uncoupled lines and linear dependent current and voltage sources is presented. Since all of these elements are available in most CAD programs used in the design of digital, as well as microwave circuits, the model can be easily incorporated in the form of subcircuits making the coupled line 2n port a standard circuit element from a computer-aided circuit analysis and design point of view.

THE MODEL

The normal mode analysis of general multiple uniformly coupled transmission lines is reasonably well known [e.g., 1-8] and is reviewed here in a simple, concise form. The voltages and currents on a lossless n-line system are described by the transmission line equations:

$$\frac{\partial v]}{\partial z} = -[L]\frac{\partial i]}{\partial t} \tag{1a}$$

$$\frac{\partial i]}{\partial z} = -[C]\frac{\partial v]}{\partial t} \tag{1b}$$

Where vectors

$$v] = [v_1, v_2 \ldots v_n]^T \text{ and}$$

$$i] = [i_1, i_2 \ldots i_n]^T$$

represent voltages and currents on the lines, T represents the transpose, and [L] and [C] the inductance and the capacitance matrices whose elements represent per unit length self and mutual parameters of the lines. It should be noted that [L] is a positive definite and [C] is a hyperdominant matrix.

Since the uniform system under consideration is time invariant, we can consider the equations in the Fourier transform or frequency domain and define V] and I] as the corresponding voltage and current vectors. Then for this exp.$[j(\omega t-\beta z)]$ variation, Eqs. (1a) and (1b) are easily decoupled leading to the eigenvalue equations for voltages and currents as given by,

$$\big[[L]\,[C] - \lambda[U]\big]\, V] = 0 \tag{2a}$$

$$\big[[C]\,[L] - \lambda[U]\big]\, I] = 0 \tag{2b}$$

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where $\lambda \triangleq \beta^2/\omega^2$ and [U] is the unit matrix.

Now, since [L] [C] and [C] [L] are adjoint, the eigenmodes of voltages and currents share the same eigenvalues (i.e., the same phase velocities) and are B-orthogonal. The above equations can be expressed as,

$$[C]\ V] = \lambda[L]^{-1}\ V] \tag{3a}$$

and

$$[L]\ I] = \lambda[C]^{-1}\ I] \tag{3b}$$

The structures of interest consist of a single or multilayered dielectric medium whose magnetic properties are the same as free space. That is, if $[C_o]$ and $[L_o]$ are the capacitance and inductance matrices for the same structure with dielectric removed then,

$$[L] = [L_o] = \mu_o\varepsilon_o[C_o]^{-1} \tag{4}$$

Equations (3a) and (3b) can be expressed in terms of capacitance matrices only by utilizing Eq. (4). They represent the generalized matrix eigenvalue and eigenvector problems of the type [A] x] = λ[B] x] found in many books on linear algebra.

Let the voltage eigenvector matrix be defined as $[M_V]$, then the current eigenvector matrix $[M_I]$ is given by [1],

$$[M_I] = [[M_V]^T]^{-1} \tag{5}$$

Substituting v] = $[M_V]$ e] and then i] = $[M_I]$ j] in the transmission line Eqs. (1a) and (1b) leads to

$$\frac{\partial e]}{\partial z} = -\mathrm{diag}[L_k]\ \frac{\partial}{\partial t}\ j] \tag{6a}$$

$$\frac{\partial j]}{\partial z} = -\mathrm{diag}[C_k]\ \frac{\partial}{\partial t}\ e] \tag{6b}$$

Where diag $[L_k]$ and diag $[C_k]$ are diagonal matrices as given by

$$[L_k] = [M_V]^{-1}\ [L]\ [[M_V]^T]^{-1} = \frac{1}{u_k^2 C_k} \tag{7a}$$

$$[C_k] = [M_V]^T\ [C]\ [M_V] \tag{7b}$$

Equations (6a) and (6b) represent n uncoupled transmission lines having equivalent capacitances and inductances as given above resulting in a mode characteristic impedance $Z_k = \sqrt{L_k/C_k}$. u_k is the phase velocity of the kth mode and is related to the kth eigenvalue λ_k. The above decoupled Eqs. (6a) and (6b), together with the orthogonality between current and voltage eigenvectors as represented by Eq. 5 , that is, the transformation according to

$$\begin{bmatrix} v] \\ j] \end{bmatrix} = \begin{bmatrix} [M_V] & 0 \\ 0 & [M_V]^T \end{bmatrix} \begin{bmatrix} e] \\ i] \end{bmatrix} \tag{8}$$

lead to an equivalent circuit model shown in Fig. 2 for a general n line system. It should be noted that Eq. (8) represents a congruent transformation and has also been implemented by an ideal transformer bank [1]. The models consist of elements represented by the relationships given by (8) at the input and output ports that are coupled to each other via the uncoupled lines represented by (6).

The model shown in Fig. 2 is a circuit that defines Eqs. (6) and (8), that is, the solution of coupled transmission line Eqs. (1a) and (1b). The model consists of uncoupled lines and linear-dependent sources and is compatible with most CAD programs including SPICE. Much of our work presented here was done on SPICE since all the model elements are available in SPICE and that it can be used to analyze multiports in both time and in frequency domain. The input parameters used for the SPICE sub-circuit (model) are:

Length of the lines.

Velocity of propagation of each normal mode of the system. $u_k = 1/\sqrt{\lambda_k}$ where λ_k is the eigenvalue of the [[L] [C]] matrix. k = 1,2,...,n.

The uncoupled normal mode characteristic impedances ($Z_k = 1/(u_k C_k)$) for each mode.

Voltage eigenvector matrix elements ξ_{ij}; i,j = 1,2,...n.

The normal mode velocities, impedances, and eigenvector matrix elements required to construct the model are derived from the capacitance and inductance matrices of the coupled line system as shown above. Several techniques are available to compute these coefficients for multiple coupled line structures in layered as well as simple medium [e.g., 6,14,15].

RESULTS

The above model can be used to analyze and formulate design procedures for symmetrical, nonsymmetrical, and multiple coupled line circuits such as four- and six-port couplers and two-port filters and transformers. The analysis and design capabilities are only limited by the CAD program being used and not the model. It should be noted that for both the frequency domain and the time domain analysis of a given analog or digital system, the coupled line multiport can be included as an integral part of the system and can be incorporated in the overall computer aided design of the system. This would be much more effective than the commonly used practice of first analyzing or designing the multiports based on idealized terminations, and then trying to assess the deviations in its performance when it is inserted in or coupled to a real system.

Several representative coupled microstrip structures have been analyzed for their frequency and time domain characteristics with linear as well as nonlinear terminations. The objectives range from an accurate representation of elements that are best represented as interconnected coupled microstrips, such as spiral inductors and interdigital capacitors, to the design of circuit elements consisting of symmetrical, nonsymmetrical, and multiple-coupled lines, such as filters and couplers. Some of these results are shown in Figures 3 through 6. These include: 1) the step response of an asymmetric coupled line four-port [9] together with its model parameters (Fig. 3), 2) the step response and cross talk in a typical symmetrical three-line six-port structure terminated in ECL OR gates (Fig. 4), 3) the frequency resonse of an edge-coupled 3 db coupler with and without velocity compensating lumped elements [16,17] (Fig. 5), and 4) the frequency response of a nominal 10 db six-port coupler (Fig. 6).

It should be noted that dedicated computer programs for the analysis of lossy as well as lossless lines applicable to linear and nonlinear terminations based on the method of characteristics [e.g.,2,3] and many others based on the immittance parameters of the 2n port are available for the design of digital and microwave circuits. The primary objective of this work is to show that most CAD programs for active, microwave and digital circuit design including SPICE for which MESFET [18] and other device models have also been reported recently, can also incorporate multiple coupled strips as a sub-circuit.

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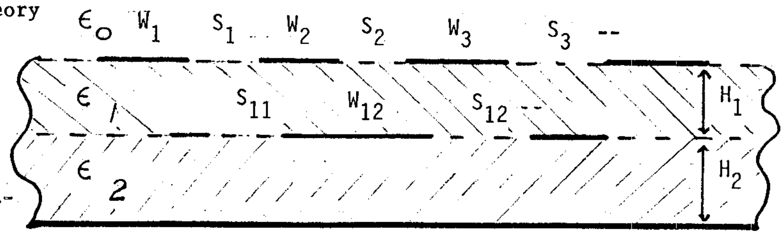

Figure 1. Schematic - cross sectional view of a multiple coupled line structure.

THE MODEL

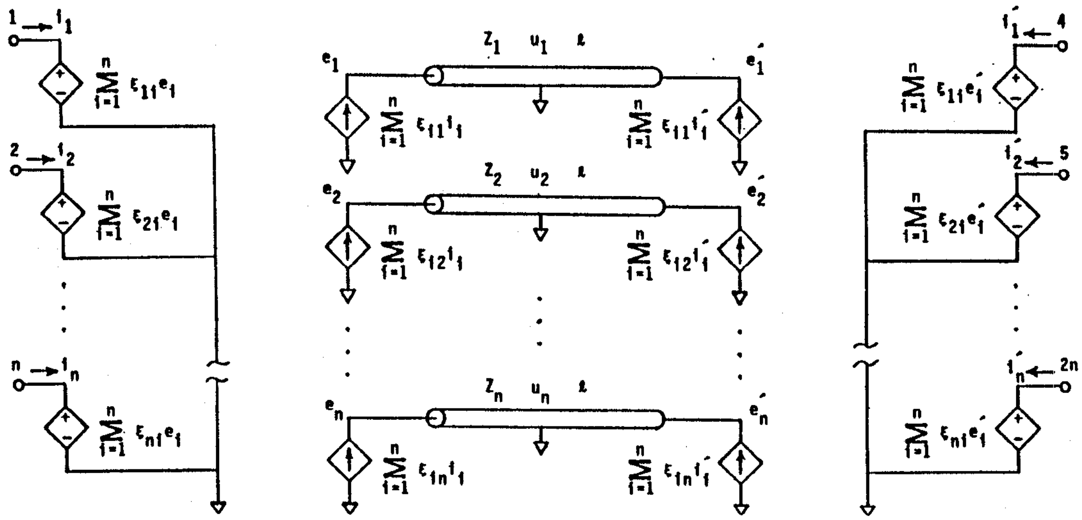

FIGURE 2

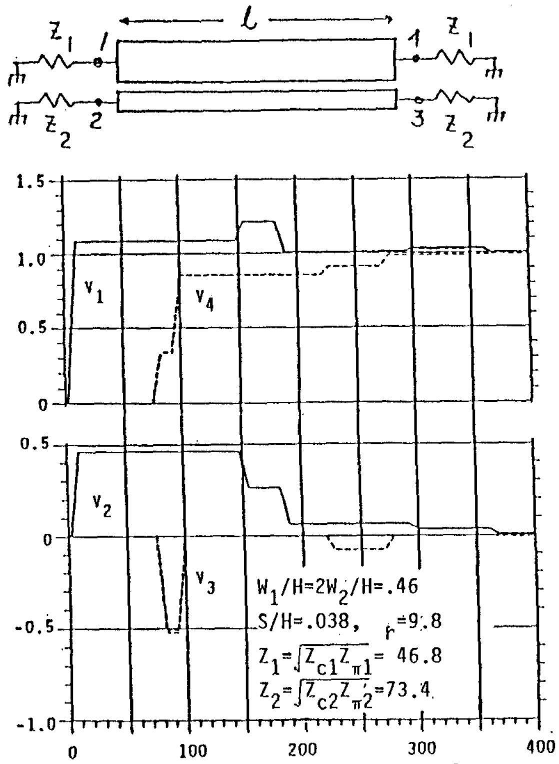

Fig. 3

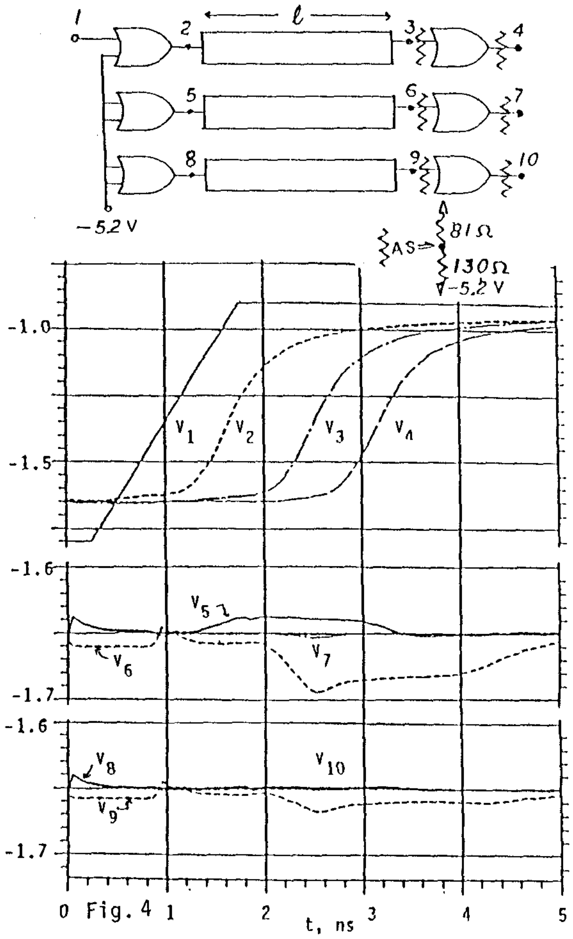

Fig. 4

Fig. 3 Step response of an asymmetric coupled microstrip four port.

Fig. 4 Step response of a three line structure on Alumina terminated in ECL OR gates. (W/H = S/H = 1.0)

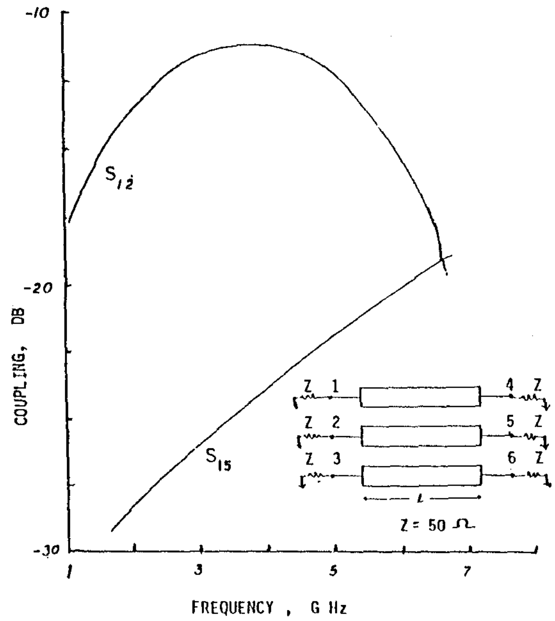

Fig.5 Scattering parameters of a three line six port 10 db (nominal) coupler.

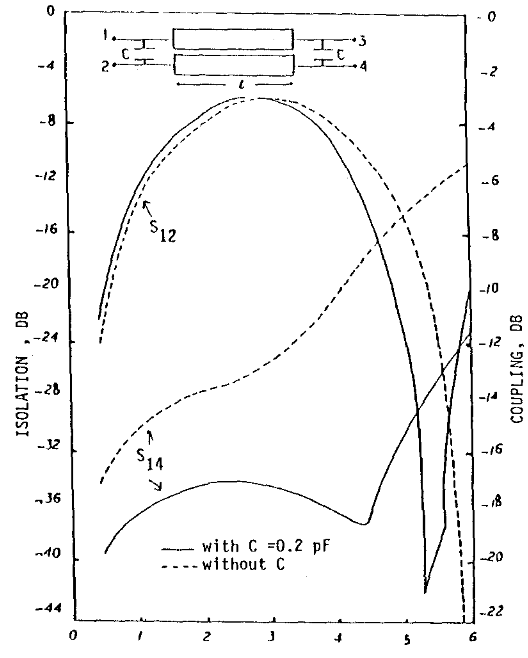

Fig.6 Isolation and coupling for an edge coupled 3 db coupler on Alumina with and without a velocity equalizing lumped capacitor[16,17]